Empirical Geostatistics #1: Kriging Slope of Regression – Sensitivities and Impacts on Estimation, Classification and Final Selection

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Abstract

With the advances in software speed and capability, many of us now are running multiple scenarios on entire models rather than just testing small areas or a few blocks. In the author's experience, both the geological and geostatistical academic theory and our rules of thumb often prove unuseable or incorrect in the real world we work in. The only way to truly validate and tune our models is to complete the entire model and 'see if it works'. If it lacks some property we were expecting, then we need to find out why and/or run different scenarios to see what changes. This is not always possible with tight project timelines, but we find ourselves doing more and more of this sort of thing, and in so doing, understand that every deposit is different and requires its own 'rules' to get valid and useful results. For the purposes of this paper, this process is termed Empirical Geostatistics.

Take Clayton Deutsch's 'all realisations all of the time' concept a step back. Before you even think of simulating, test the alternate realisations generated by alternate parameters such as different domaining, varying search neighbourhood parameters etc.

Topic #1: Kriging Slope of Regression

Never has a statistic been more misunderstood, misused and abused than the poor old kriging slope of regression.

- Maximise me! (no, you're an oxymoron)
- Classify me – I don't care how!
- Block size with me – bigger is always better (really?)
- Drill space with me (but please don't de-cluster me)
- Don't top cut me, threshold me (actually it doesn't matter – or does it?)

This paper takes a fresh look at the humble kriging slope of regression (KSOR) and its close friend, kriging efficiency (KE), examines and shows examples of associated misuse and misunderstanding, while tying these back to potential impacts on classification, grade and tonnage estimates and final selection outcomes.

Keywords: kriging slope of regression, kriging efficiency, kriging neighbourhood analysis, conditional bias

Introduction

The use of the KSOR is intimately tied up with the historical elimination of conditional bias debate. Historical because, in the author's opinion, the debate is now over within the geostatistics community. Despite this, the message still has not penetrated down to the everyday practitioner, the Resource geologist, despite having been articulated over 25 years ago.

The majority of software packages that contain tools for kriging neighbourhood analysis (KNA) still refer to maximising the KSOR or KE as the desired target. This is misleading at best, and in some circumstances, completely incorrect.

The misuse of the KSOR arises from a fundamental misunderstanding of the desired use of any particular block model estimate. The different uses can be described in a number of different ways. Global accuracy versus local accuracy. Long-term planning versus short-term planning. Prediction versus performance.

The KSOR discussion is also intimately linked with the small block discussion. Historically, at the practitioner level, this obsession with elimination of conditional bias and absolute requirement for local accuracy (implemented by the maximisation of the KSOR), was driven by Krige, (most notably in a 1996 paper, but consistently over his lifetime), by Armstrong and Champigny's 1989 paper on the dangers of small block kriging and then in the 2003 paper by Vann, Jackson and Bertolli extoling the supposed virtues of Kriging neighbourhood analysis (KNA) and KSOR maximisation.

In fact, maximisation of the KSOR and elimination of conditional bias are desirable only if the model is designed for final selection. That is, the model will be used to physically mark out on/in the ground what is to be mined and processed, and what is to be rejected as waste.

Both theory and practice have shown that large block, highly smoothed estimates that result from maximisation of the KSOR or KE during KNA are often totally inappropriate for financial studies and long-term planning because, under typical Resource stage drilling information levels, they can severely distort the global grade tonnage curves in comparison the grade and tonnage curves that result from models created with significantly more information at the grade control stage.

THE KRIGING SLOPE OF REGRESSION

What is it and how is it calculated?

Conceptually, the KSOR is the slope of the linear regression between true and estimated block values over a domain (Figure 1). We can never know the true estimate, however, there are cases where we can consider an estimate carried out with dense data, such as a grade control model, as 'True' when compared to a model estimated with fewer data, such as a Resource model from exploration drilling. This is a somewhat reductive argument because, in fact, the grade control model is not the truth either, simply a better estimate than the Resource model because it is based on significantly more information.

At the practical level, KSOR is a mathematical construct calculated at each block, which is a function of the variogram model and the local block/sample geometry. Refer to Chiles & Delfiner 1999 (p163), Deutsch 2007 and many others for calculation details.

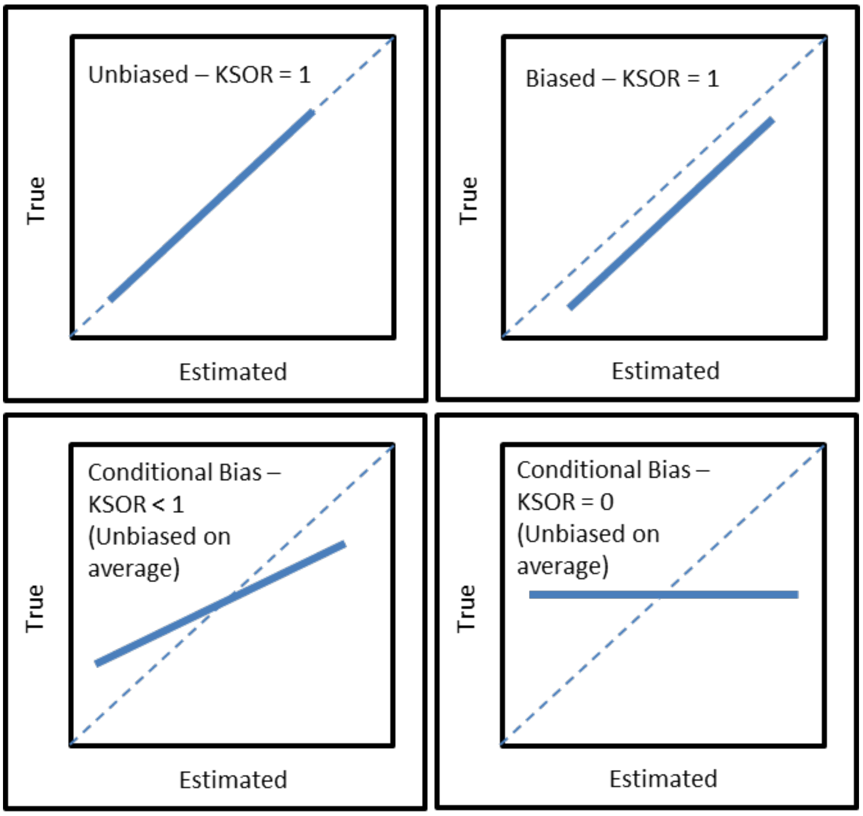


Figure 1: KSOR and bias

What does it mean?

If the KSOR is equal to one, an estimate is said to be conditional unbiased, which means; that minimsation of error variances between true block values and estimated block values has been achieved on average; block estimates within a domain should on average display no bias during final selection at all cut offs; and that the unavoidable misclassification (Figure 2) will be balanced and the material correctly classified above any cut off is maximised. The key emphasis here is, 'during final selection'.

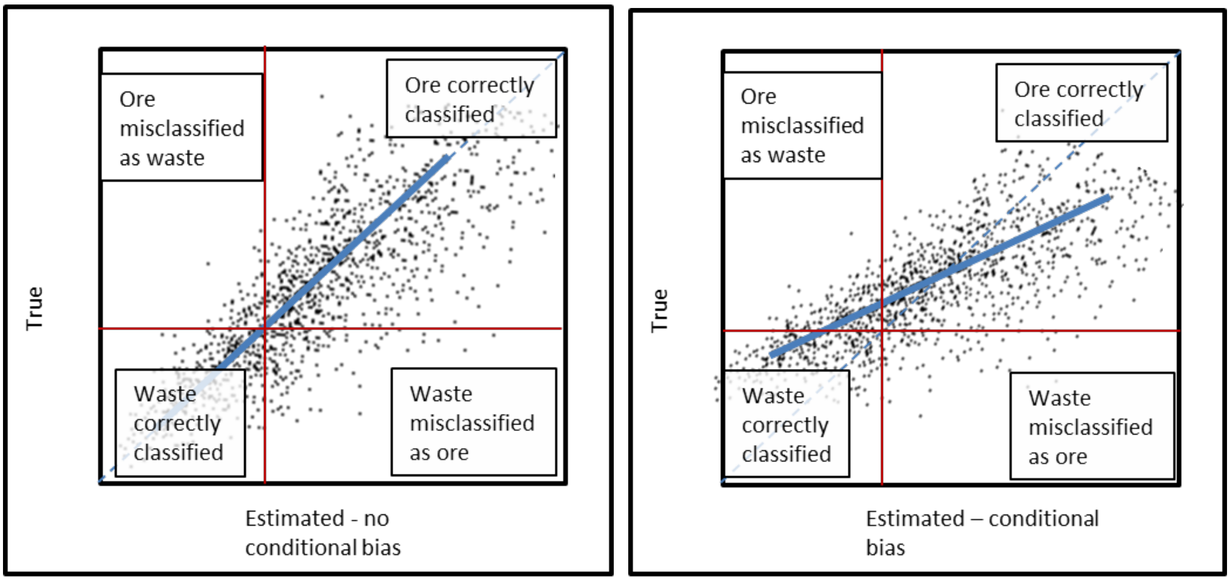


Figure 2: Misclassification diagrams

What is it sensitive to?

The KSOR is dependent on many parameters; the block size, the block discretisation, the variogram model and the search neighbourhood parameters (Figure 3). With the exception of the variogram, these are all the parameters that are examined in what has come to be known as Kriging Neighbourhood Analysis (KNA). The kriging neighbourhood is also referred to in some texts as the Kriging plan, the search neighbourhood, or the moving neigbourhood. Interestingly, the KSOR is not sensitive to the actual sample values within any particular Kriging neighbourhood (with the exception of the fact that the sample values are used to derive the variogram model).

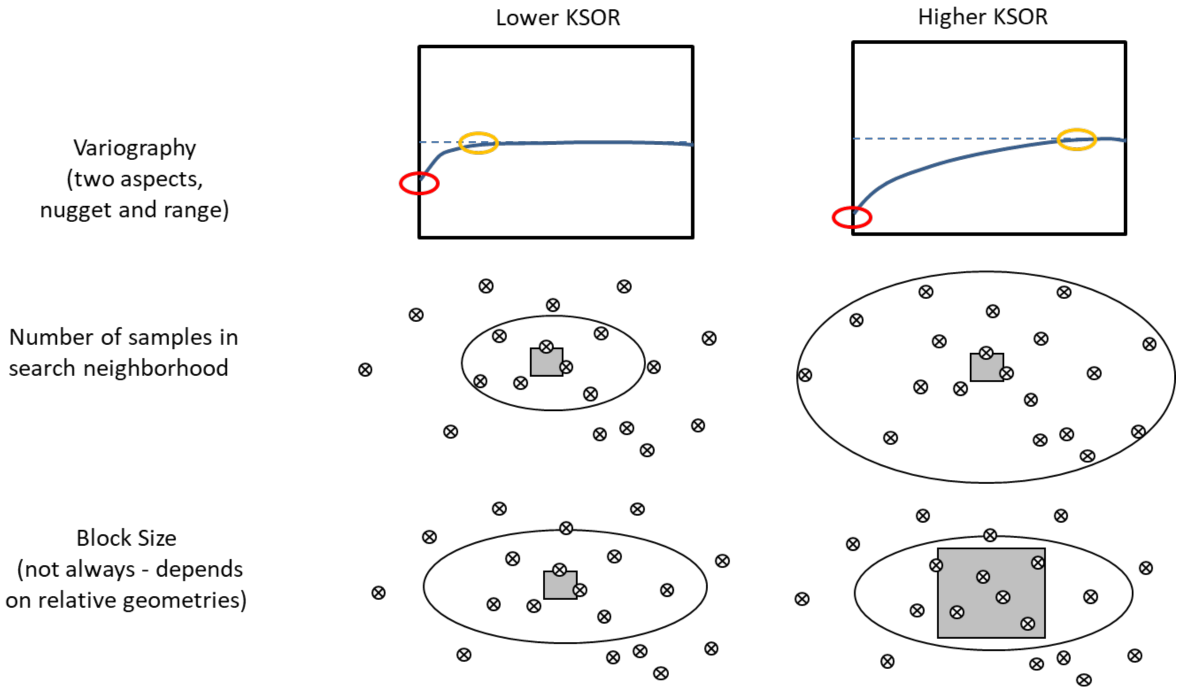


Figure 3: KSOR sensitivity

The relationship between kriging variance, kriging regression slope and kriging efficiency

The KSOR, KE and kriging variance are closely related. Mathematical definitions of the relationships can be found in Krige 1996, Novak and Leuangthong 2016 and many others. Some more practical examples of particular real data sets are shown in the figures 4 and 5.

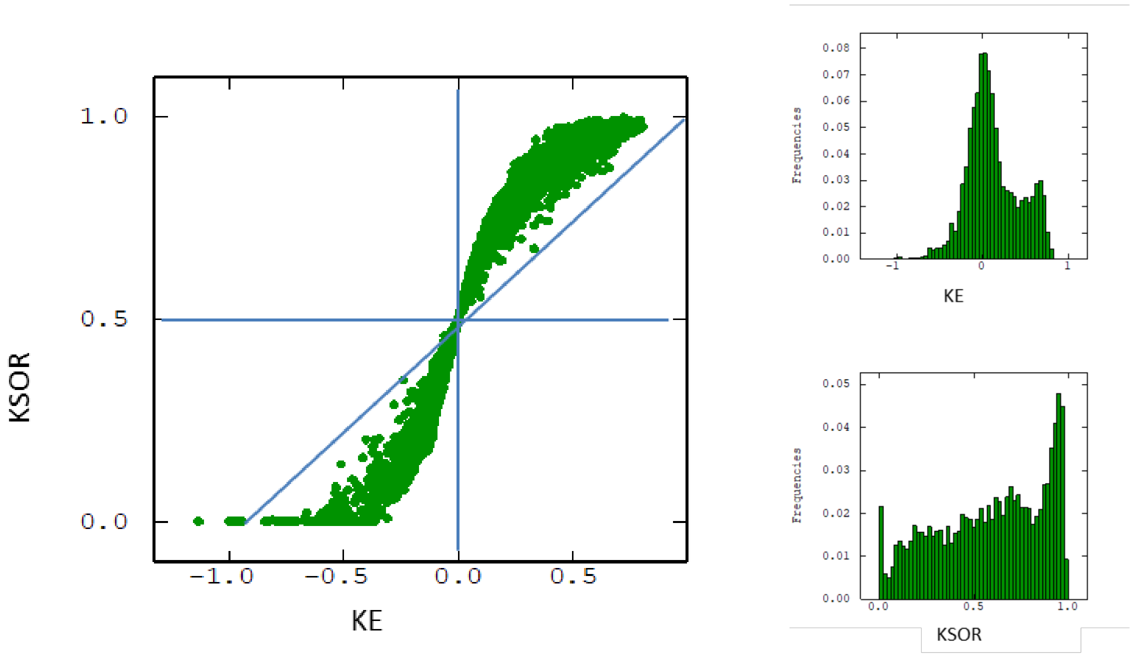


Figure 4: An example of the relationship between KSOR and KE

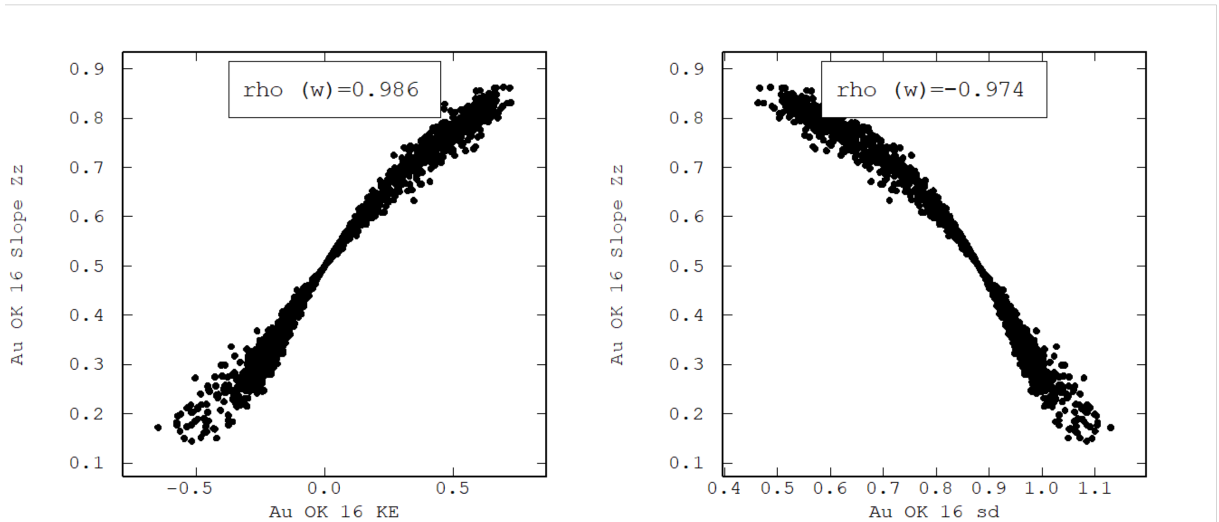


Figure 5: An example of the relationship between KSOR and KE and Kriging variance (presented as standard deviation in this case)

History

A concise quote from Chiles and Delfiner makes an appropriate opening statement for this section. 'What is the optimum design of a moving neighbourhood? This question turns out to be rather complex. Short of a rigorous theory we can only give some guidelines.'

The KSOR is a by-product of all the choices we make as we set up the block estimation process. In some cases, such as KNA, we may also use it as a guide to tune our choices. Many practitioners will go on to use the KSOR, in whole or in part, as part of their classification system. What exactly are the impacts of these choices on the KSOR and what does this mean for local and global accuracy of our estimates, both at Resource definition (mine planning) and grade control (final selection) stages of a project?

An extensive, but illuminating, selected history of the issues and views on conditional bias, the KSOR and KNA is given here in a timeline of the literature with selected quotations. This history has been given in such extensive detail in this paper because, it provides more context and understanding of the issues around the KSOR than any technical discussion or practical examples.

Timeline

Krige 1951 – paper – A Statistical approach to some basic mine valuation problems on the Witwatersrand

'… it is obvious that block valuation based on a limited number of samples per block will result in the general under-valuation of blocks listed in the low grade categories and over-valuation of blocks listed in the high grade categories.'

Matheron 1963 – paper – Principles of geostatistics

'However appreciable they are, the improvements of accuracy provided by the kriging would not always justify the amount of calculations it requires. In most cases, the major interest of the procedure does not come for the reduction of estimation variances but from its being able to eliminate the cause of systematical [sic] error.'

David 1977 – textbook – Geostatistical Ore Reserve Estimation

'… it is not possible to know at the same time how much ore one will have within a given block and where it is. We can predict the frequency distribution of the ore grade but we cannot localise the blocks, or we can predict as well as possible the grade of blocks, precisely located, but the frequency distribution will be smoothed. Let us say once more that if no further information is acquired the last method is adequate. If we try to see why planning people ask for very small block predictions, it is found that this is most of the time based on the wrong assumption that, the smaller the blocks, the better the knowledge of the deposit. If one now looks at what is important for a mine planner, one sees that planning is made in terms of broad blocks only, which have to be mined, whatever they are found to contain, as the excavation has to proceed. Consequently, what is really needed is only the grade of rather large blocks and within these blocks, the proportion of ore and waste. Which small blocks in the large one are really ore and which are really waste is totally irrelevant for monthly planning, less to say for quarterly or yearly planning as long as their percentages of occurrence can be predicted.'

Journel & Huijbregts 1978 – textbook – Mining Geostatistics

'A prime objective of defining a data neighbourhood [A(x0)] should be to ensure that it avoids all risks of bias in the estimation. Thus, in two dimensions, for example, when the data configuration is such that one direction [B] is under sampled, although the structural continuity in this direction is significant, it is advisable to extend the estimation neighbourhood [A(x)] in this direction so as to include more data which will take into account the [B] directional continuity in the estimation. Examples of such a procedure are the automatic mapping programs that partition the space around [x0] into equal parts (eg. four quadrants or eight octants) and only the two or three data closest to [x0] are considered within each of these parts.'

Parker 1979 – paper – The volume variance relationship: A useful tool for mine planning

'Best practice for interim estimates is to use a restricted search with ordinary kriging to compute estimates that closely matches the anticipated grade tonnage curve.'

Matheron 1983 – paper – The selectivity of distributions and the second principle of geostatistics

'In any case, we must expect that an increase of the size of support or decrease of the ultimate information will result in a distortion or adulteration of the true grade tonnage curves. And very often this distortion will be much more important than any estimation error.'

Guibal & Renarcre 1983 – paper – Local estimation of the recoverable reserves: Comparing various methods with the reality on a porphyry copper deposit

'The general problem comes from the fact that the characteristics of many deposits are incompatible with the requirements of non-linear methods (i.e. strictly stationary hypothesis).'

Rivoirard 1987 – paper – Two key parameters when choosing a Kriging neighbourhood

'In the stationary case, two parameters are especially interesting when choosing the kriging neighbourhood: weight of the mean, which shows how kriging depends on the neighbourhood, and slope of the regression, which indicates if the neighbourhood is large enough'

Armstrong and Champigny 1989 – paper – A study on kriging small blocks.

'… it is seen that when the range is 30 m (i.e. much larger than the grid spacing), all the block estimates are reasonably well correlated with their actual values. Moreover, the slope of the regression is close to 1.0 in all cases. This means that the kriged block estimates can be used for selecting ore blocks from waste. However, if the range is only 10 m (half the grid spacing), the four corner blocks are reasonably well estimated but the others are not. The regression slope and the correlation coefficient are zero for the central blocks. The kriged estimate for the central blocks is just an estimate of the local mean. This is the best estimate of their grades given the information but it would clearly not be wise to use this value to predict whether the blocks are ore or waste.'

Isaaks & Srivastava 1989 – textbook – An Introduction to Applied Geostatistics

'Having chosen an orientation and anisotropy for our search ellipse, we still have to decide how big to make it. The simple answer is that it must be big enough to contain some samples…In practice one tries to have at least 12…Some practitioners make maps of estimates in typical areas using various search strategies. Estimates are first made using a large number of samples. Then the search strategy is changed to reduce the number of samples and the corresponding estimates mapped. The search strategy is deemed appropriate just before the estimates begin to show noticeable differences with less samples.'

Krige 1996 – paper – A practical analysis of the effects of spatial structure and of data available and accessed, on conditional biases in ordinary kriging

'Ore valuation for a new mining project or an existing mine basically covers two major stages. At the initial or first stage the data is limited and is obtained either from a broad drill hole grid or from the initial main development grid. During the second or final stage more data becomes available from a closer drill hole or blast hole grid or from sampling of stope faces and auxiliary development; this is also the stage of final selection of blocks as ore (payable) or waste (unpayable)…At both stages of valuation mentioned above, individual block valuations will be subject to error due to the data limitations. The estimated error levels can provide a basis for classification of the reserves into the required categories. Therefore, the valuation technique used should ensure minimum error variances and this will be the case if the appropriate kriging and data search routines are used. These requirements are linked closely to the expected slopes of regression of the eventual follow-up values, (usually inside the blocks) on the original block estimates. Slopes of less than unity indicate the presence of conditional biases with blocks in the upper grade categories overvalued and the reverse applying to blocks valued as low grade…Various attempts have been made to reduce or eliminate the 'smoothing' effect but this can only be achieved at the expense of introducing conditional biases in the individual block valuations. Such a practice is completely unacceptable…Where the effects of smoothing is expected to be significant at the first stage, early production and financial planning can be based on global adjustments to tonnages and grades. However, individual block estimates cannot be adjusted but can either be qualified by estimates of recoverable tonnages and grades for each block or, alternatively, mine planning and financial studies can be performed on a series of acceptable simulations in order to define the overall levels of uncertainty.'

Goovaerts 1997 – textbook – Geostatistics for Natural Resources Evaluation

> ‘In Practice 10 data values would be a reasonable minimum. The number should be larger where data are clustered so that one or two of the nearest data do not screen all others. The maximum number of data values to retain depends on the objective pursued. When one aims at depicting local features of the attribute, that number should be limited; whereas more data and data farther away should be retained to depict long range structures.’

Armstrong 1998 – textbook – Basic Linear Geostatistics

> ‘Consequently, the grade and tonnage curve calculated from the kriged blocks is quite different from the real one. This shows that kriging should not be used for estimating the grades of small blocks from widely spaced data.’

Chiles and Delfiner 1999 – textbook – Geostatistics : Modelling Spatial Uncertainty

> ‘In theory, the minimised mean squared error is achieved when all points are included, ...but a global neighbourhood may result in a kriging matrix which is too large to be inverted numerically. Typical algorithms work well up to around 100 points. Another consideration is that the geostatistical model itself may only be valid over short distances. ...What is the optimum design of a moving neighbourhood? This question turns out to be rather complex. Short of a rigorous theory we can only give some guidelines. …following a strategy that attempts to sample all directions as uniformly as possible (Octant search). ..Typically, from 8 to 16 points are retained…’

Olea 1999 – textbook – Geostatistics for Engineers and Earth Scientists

> ‘The second and more serious drawback of dealing selectively with large samplings is the lack of tests for determining appropriate neighborhood size. In very general terms, the neighborhood must be large enough to contain three observations at a bare minimum and anything beyond 25 observations is considered more than adequate (Myers, 1991). More precise justification for selection of neighborhood size depends upon the measure of performance achieved and fluctuates according to the nature of the sampling pattern and the covariance. Lacking theoretical criteria, experimentation such as that presented in [Internal reference] is the best alternative.’

McLennan & Deutsch 2002 – paper – Conditional bias of geostatistical simulation for estimation of recoverable reserves

> ‘There are two schools of thought related to the conditional bias and smoothing of ordinary kriging for mine planning: 1. The ‘conditional bias of block estimates is always wrong’ school…Here, one never accepts block estimates known to be wrong in expected value. Large search routines retaining many conditioning data are implemented to minimize uncertainty and conditional bias. The price, however, is block estimates that are smooth and near the mean. 2. The ‘let’s get recoverable reserves right’ school…The idea here is to anticipate the dispersion variance of the true block grades. Fewer samples are used in the kriging plan to increase the variability of the block estimates in the hope of reproducing the true block grade dispersion variance. The price of this approach, however, is block estimates that are conditionally biased.’

Sans et. al. 2002 – paper – Conditional simulation at Phosphate Hill

> ‘The kriging procedure needs to be carried out with a kriging neighbourhood that is optimised so that the unavoidable degree of smoothing remains manageable, and the equally unavoidable conditional bias in the kriging estimation is well controlled and acceptable. This implies that the kriging is completed in such a way that an acceptable balance is achieved between the degree of smoothing (due to the averaging out of sample values) and the level of conditional bias.’

Van Jackson & Bertoli 2003 – paper – Quantitative kriging neighbourhood analysis for the mining geologist – A description of the method with worked case examples

> ‘Ideally, the slope of the regression a should be very close to 1.0 and thus imply conditional unbiasedness. In these circumstances, the true grade of a set of blocks should be approximately equal to the grade predicted by the kriged estimation.’

Isaaks 2003 – The Kriging oxymoron: A conditionally unbiased and accurate predictor (2nd edition)

> ’A conditionally unbiased and accurate predictor is an oxymoron. The estimator for a long-term mine planning block model may be conditionally unbiased but then the histogram of block estimates will be smoothed yielding inaccurate predictions of the recoverable tons and grade above cutoff grade. Conversely, if the histogram of block estimates provides accurate predictions, then the block estimator is necessarily conditionally biased. The estimator for a long-term mine planning block model cannot be conditionally unbiased and simultaneously accurate.”

Abzalov 2006 – paper – Localised uniform conditioning: A new approach for direct modelling of small blocks

> ‘The advantage of this approach is essentially dependent upon the data available for ranking the small blocks within a panel in increasing order of their grade. Ordinary Kriging of the small blocks can be used for their ranking providing the kriged estimates produce a meaningful indication of the relative grade pattern. Where the data is sparse and not close to a panel, or their distribution is characterised by a strong short-range variability, the advantages of using the Localised Uniform Conditioning approach are more limited.’

Deutsch 2007 – paper – The slope of regression for kriging estimators

> ‘Our concern with the slope of regression is conditional bias…Conditional bias is a serious problem if the estimate is going to be used for a final or near-final decision, for example, for grade control in open pit or for stope estimates in underground. It would be a serious mistake to have a known bias in estimates used for final decision making. On the other hand, we may be interested in estimates for interim planning purposes, that is, final estimates will be calculated in the future with additional information. We may accept conditional bias in interim estimates if the estimates have more desirable properties. The most common desirable property to have is a reasonable estimate of global reserves. Kriging with sparse data will lead to estimates that are overly smooth. A greater amount of higher and lower final estimates will be calculated when the final information is obtained. Thus, it may be a serious mistake to use smooth conditionally unbiased interim estimates for planning; we should anticipate the information available in the future. In general, there is no universal best estimator. ‘best’ must be defined for each situation. The debate should turn from conditional bias to the purpose of the estimate and the goals of the study.’

Rossi and Deutsch 2014 – textbook – Mineral Resource Estimation

> ’There is a direct relationship between the number of samples used and the conditional bias vs recoverable resources accuracy debate…All of these [search neighbourhood] parameters can be modified to a certain extent to obtain a resource model that achieves specific goals. Ideally the process of setting up [search neighbourhoods] becomes iterative because some kind of calibration procedure is being used. The type of calibration that can be used depends on whether the mineral deposit being estimated is in production or not.’

Kentwell 2014 – paper – Aligning resource estimates with mine planning

'…the current industry standard default practice of targeting maximum individual block accuracy (conditional unbiasedness) using OK at resource stage, without consideration or evaluation of the extent of associated smoothing at SMU scale, can lead to significant errors in fundamental mine planning decisions. This then has implications for resource classification if the classifications are based heavily on the OK quality without consideration of the true SMU distribution.'

Deutsch and Deutsch 2015 – lesson – Introduction to choosing a Kriging plan

'We classify the purpose of estimates into four categories: 1) interim estimates, 2) final estimates, 3) visualization and trend models, and 4) probabilistic predictions. Each of these estimation purposes has a different set of criteria for choosing the best kriging search plan…Although the slope of regression is often documented, it is of little utility when choosing a kriging search plan. In an interim estimation context, the most important parameter is matching the desired histogram…Application of these efficiency measures (KSOR and KE] for decision making is challenging due to large differences between mineral deposits.'

Nowak & Luangthong 2016 – paper – Conditional bias – let's keep it!

'In short, optimising kriged block estimates with slope of regression or kriging efficiency measures may lead to block models that do not adequately reflect true block grades. It is tempting and easy to use slope of regression and kriging efficiency for validation of block estimated grades. Those measures are commonly available in commercial software packages. Although theoretically high slope of regression, i.e. low conditional bias, is considered necessary for good quality estimates, in practice this approach may be outright harmful if the objective of the study is to predict global resource quantities above an economic cut off grade. Both measures are a reflection of a modelled variogram and data locations and do not take into account actual assay values, or their variability in the vicinity of an estimated block. Moreover, it is often quite difficult to construct a reliable variogram model, particularly in early exploration stages, and relying on its metrics to design 'best' resource estimates cannot be considered best practice.'

Software

Many of the major mining and geostatistics packages state that targeting a KSOR and KE value of 1 is the desired goal of KNA. The author did not find any discussion of local versus global entered into in any of the major mining and geostatistics packages help files. Is it any wonder that the KSOR is misunderstood?

Examples/case studies

Local versus global accuracy at planning stage (prediction)

We will examine three separate real data sets which cover three different situations that are encountered during estimation.

The first case is that of a nickel laterite domain where average drill spacing ranges between 5 m and 15 m, the block size is 5 m x 5 m x 4 m, the variogram nugget is approximately 35% and anisotropic variogram ranges are between 50 m and 150 m depending on direction. We will call this case the Abundant Information case.

The second case in an open cut gold example where average drill spacing is 25 m x 12.5m , the block size is 25 m x 12 m x 5 m, the variogram nugget is 55% and the anisotropic variogram rangers are between 10 m and 50 m depending on direction. We will call this case the Typical Information case.

The third case is an underground gold vein style domain where the drill spacing is highly variable ranging between 5 m and 50 m, the block size is 5 m x 5 m x 10 m, the variogram nugget is approximately 45% and the anisotropic variogram ranges are between 5 m and 25 m depending on direction. We will call this case the Minimal Information case.

For each case a number of estimates have been run using increasing maximum sample numbers. In some cases, a global theoretical change of support (TCOS) and/or grade control model are also available.

Note that in the sections below a conventional profit curve (or profit curve) is defined as:

Conventional Profit = Profit metal = tonnage above cut off * (grade above cut off – cut off grade).

The Abundant Information case

For the Abundant Information case we can see that globally there is essentially no difference in the grade, tonnage, metal and profit curves (Figure 6) for any of the estimates and that the estimates are a very close global match with the theoretical change of support curves. However, locally there are differences at the block scale (Figure 7).

Table 1 shows that although there is no discernible difference in the curves, locally, the KSOR and KE continue to improve with increasing sample numbers and that the theoretical maximum KE of 1 is still a long way off.

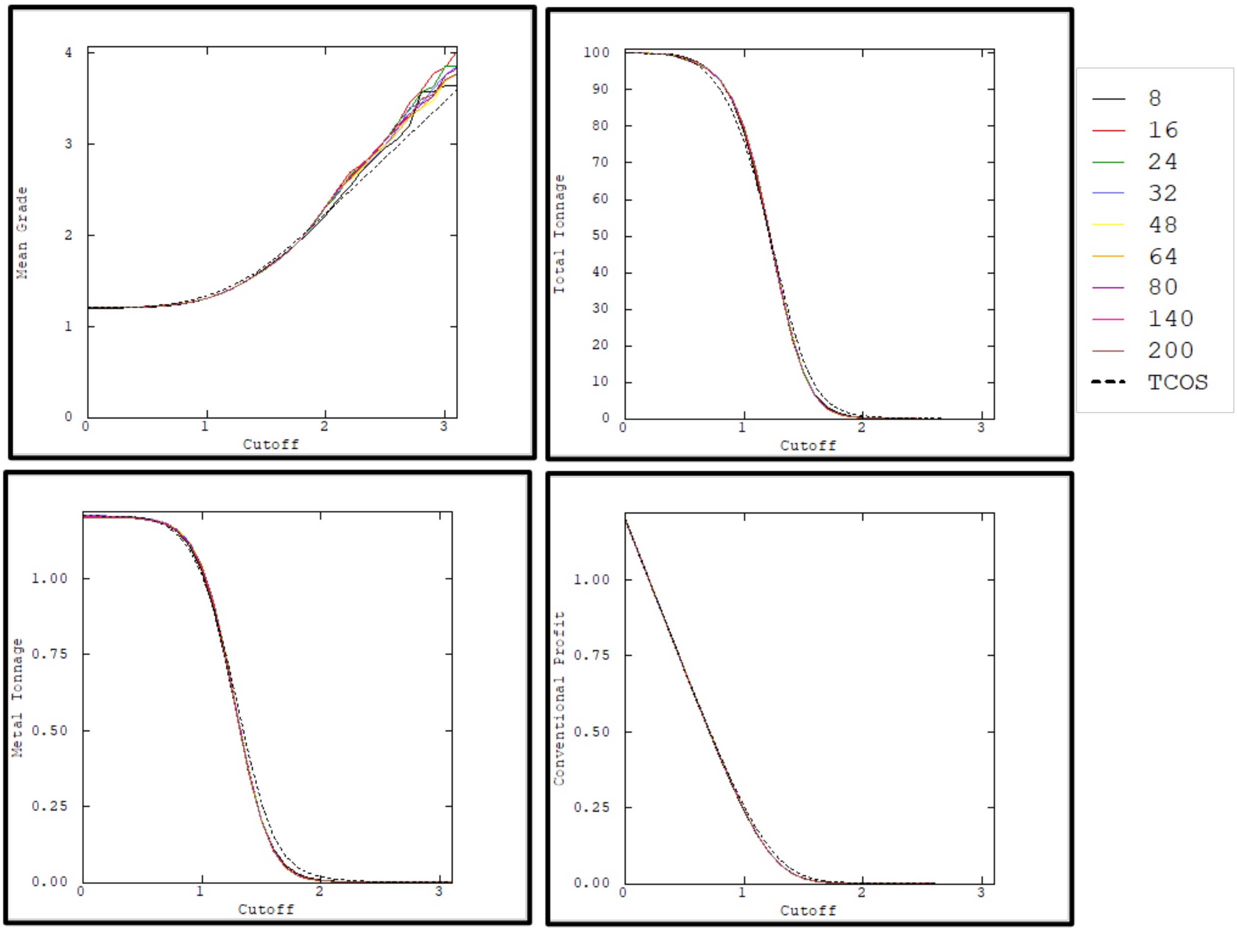


Figure 6: Grade, tonnage, metal and profit curves for a range of estimates – block size similar drill spacing, low nugget long range – KSORs between 0.7 and 1.0

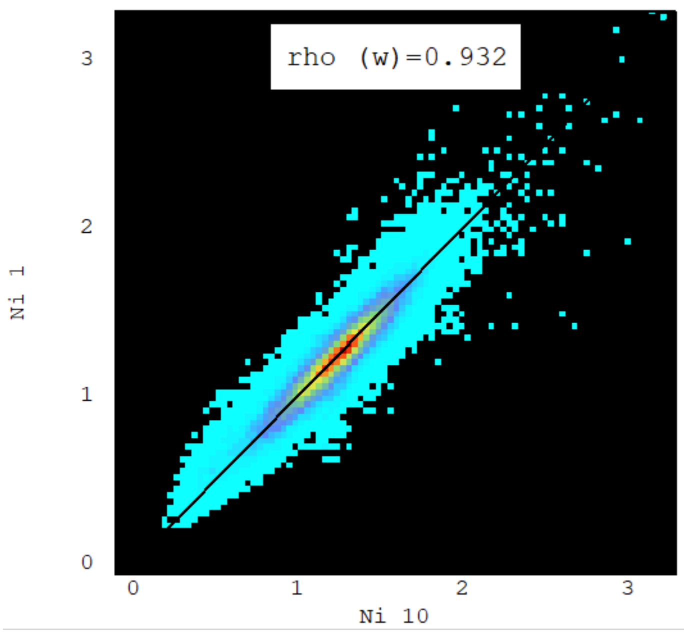


Figure 7: Abundant Information case scatterplot of 80 vs 4 sample estimates.

Maximum Samples	Estimation Block Mean	Estimation Block Variance	Sum of the Positive weights	KSOR	KE	Kriging Variance
4	1.19	0.09	1.00	0.75	0.47	0.072
8	1.20	0.08	1.00	0.86	0.55	0.062
16	1.21	0.08	1.00	0.92	0.62	0.052
24	1.21	0.08	1.01	0.94	0.64	0.048
32	1.21	0.08	1.01	0.95	0.65	0.044
48	1.21	0.08	1.04	0.97	0.66	0.044
64	1.21	0.08	1.06	0.97	0.67	0.044
80	1.20	0.08	1.08	0.98	0.67	0.044
120	1.20	0.08	1.11	0.98	0.67	0.044
160	1.20	0.08	1.14	0.98	0.68	0.044

Table 1: Global statistics for the Abundant Information case estimates

The Typical Information case

For the Typical Information case we can see that different sample number curves are spread (Figure 8). Higher sample number estimates globally display lower grades and metal above cut off. Higher sample number estimates globally show higher tonnages at cut offs below the mean and lower tonnages at cut offs above the mean. Most interestingly, higher sample numbers display globally lower conventional profit. Although difficult to see in the figures, the 8-sample estimate is closest to the theoretical change of support curves. Most strikingly, the 4-sample estimate is globally closest to the grade control model.

For the Typical Information case, locally there is a large scatter between the low sample and high sample estimates (Figure 9). Comparison with the grade control model at the local block by block level shows that the low sample estimates are very poorly correlated whereas the higher sample (smoothed) estimates show much better correlation (Figure 10).

Table 2 shows the domain average KSOR and KE result for the increasing sample number estimates as well as for the grade control model. The actual regression slopes with the grade control model are also shown (Scatterplot SOR against grade control).

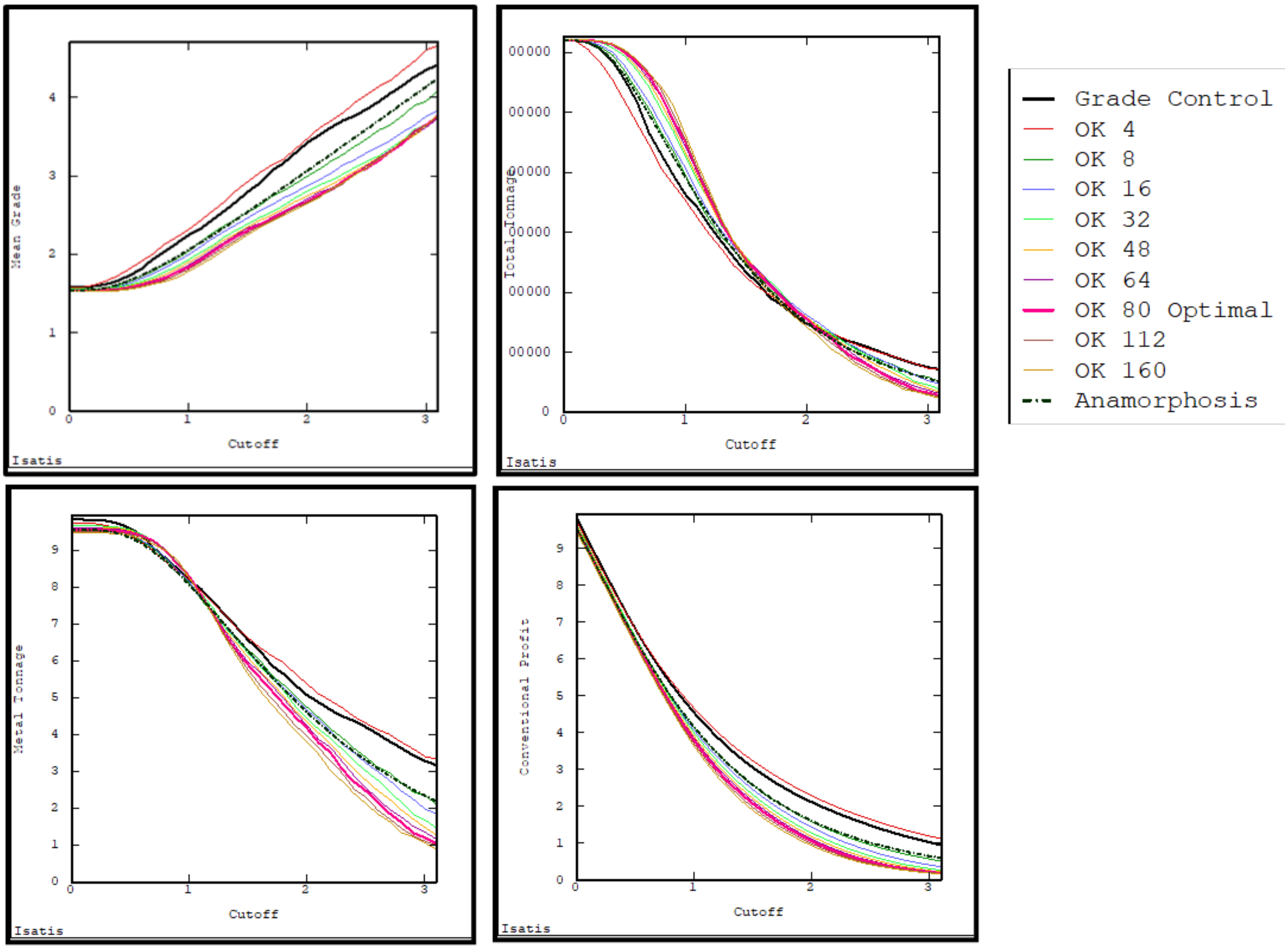


Figure 8: Grade, tonnage, metal and profit curves for a range of estimates – block size similar drill spacing, medium nugget medium range – KSORs between 0.3 and 0.9

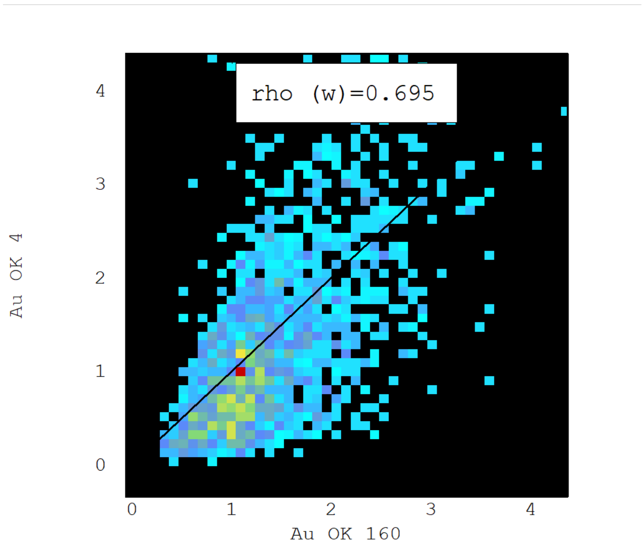
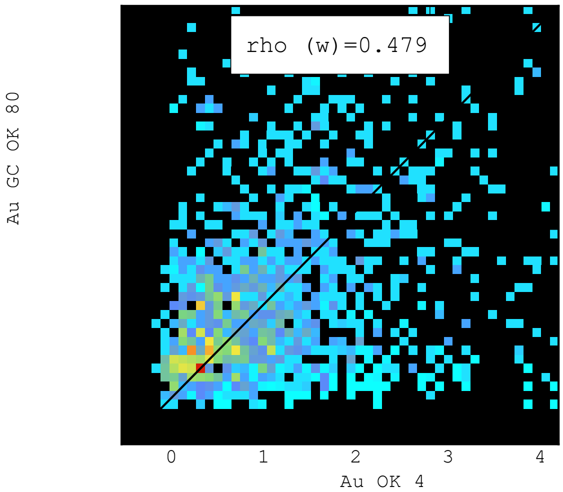


Figure 9: Typical Information case scatterplot of 160- versus 4-sample estimates



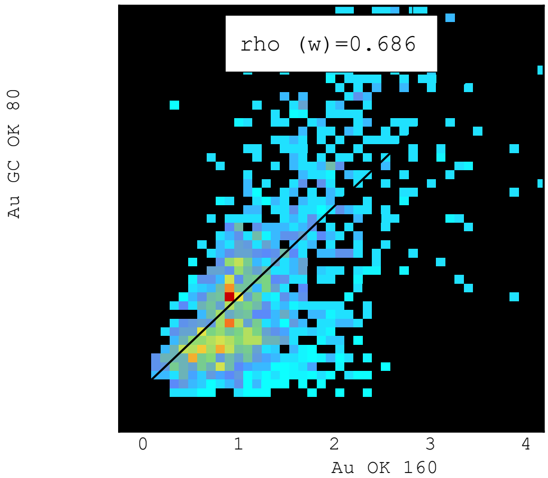


Figure 10: Scatterplot of 4-sample and 160-sample estimates against grade control

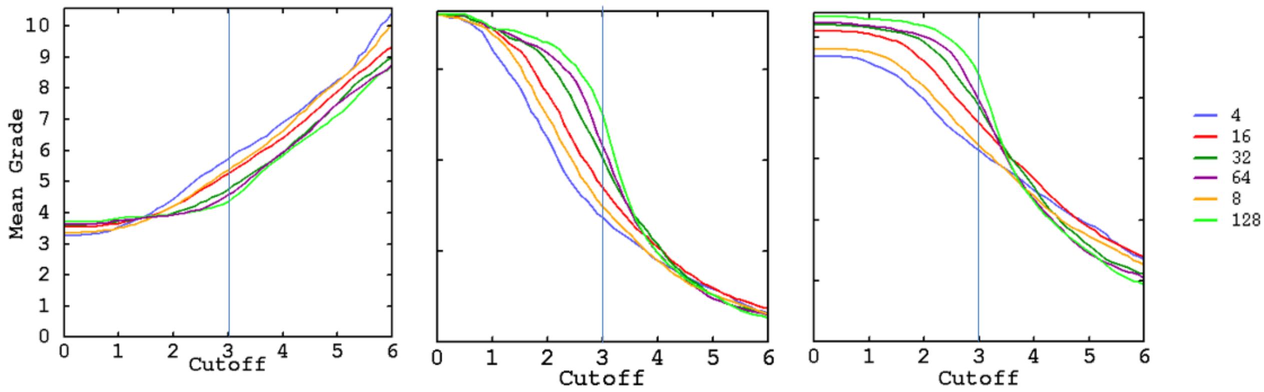
Method	Nominal Maximum Samples	Average Samples	Estimation Block Mean	Estimation Block Variance	Sum of the positive weights	KSOR	KE	Scatterplot SOR estimate vs grade control
GC OK 80	80	76	1.58	1.67	1.00	0.89	nr	na
OK	4	4	1.51	1.72	1.00	0.34	-0.44	0.47
OK	8	11	1.49	0.96	1.00	0.53	0.09	0.74
OK	16	14	1.49	0.83	1.00	0.57	0.16	0.84
OK	32	27	1.50	0.69	1.00	0.69	0.29	0.98
OK	48	39	1.49	0.63	1.00	0.76	0.34	1.08
OK	64	51	1.49	0.60	1.00	0.80	0.37	1.12
OK	80	63	1.49	0.59	1.00	0.83	0.38	1.14
OK	112	85	1.49	0.57	1.01	0.86	0.39	1.17
OK	160	115	1.50	0.55	1.02	0.89	0.40	1.20

Table 2: Global statistics for the Typical Information case estimates

The Minimal Information case

For the Minimal Information case, globally, the curves show very large differences between low and high sample number estimates (Figure 12). Although not shown in the figures, the 16-sample estimate is globally closest to the TCOS curves. Locally, again there is a large scatter when comparing the high and low sample estimates against each other.

As an exercise in extending the theory to its logical extreme, an estimate was run with a unique neighbourhood (all samples in the domain are used estimate every block). This produced some interesting results (Figure 13). Firstly, the estimated mean now becomes the un-declustered sample mean, which for this mixed drill spacing domain, is significantly different to the de-clustered sample mean. Secondly, the scatterplot between the 16-sample estimate (as the estimate nearest to the TCOS estimate) and the unique estimate shows the tendency of ‘regression to the mean’ (Figure 14) of the logical outcome of maximising sample numbers and KSOR for poorly informed domains. Note that even with all samples in the Minimal Information case, the KE is barely above zero (Table 3).



Grade | Tonnage | Metal

Figure 12: Grade, tonnage and metal curves for a range of estimates – small block wide spaced drilling, medium nugget short range - KSORs between 0 and 0.1

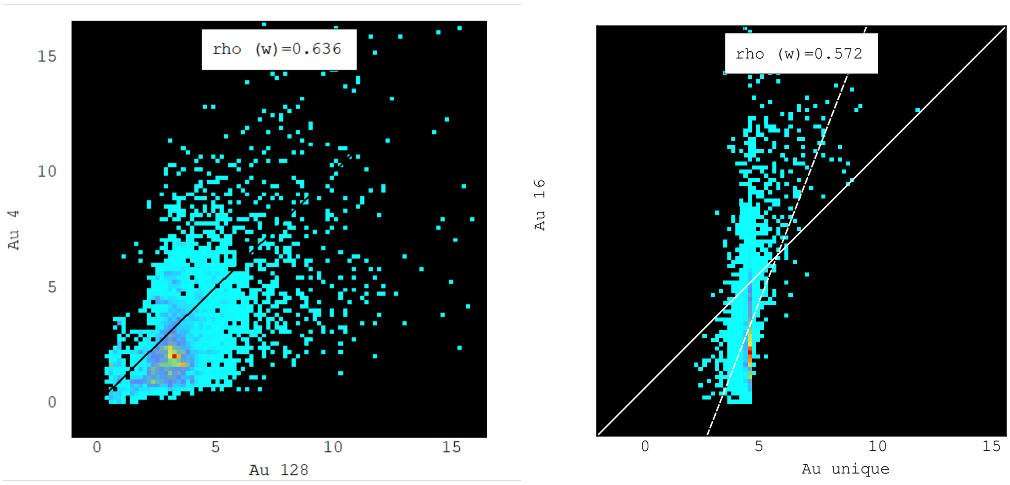


Figure 13: Minimal Information case scatterplot of 128 versus 4 and unique versus 16-sample estimates.

Maximum Samples	Estimation Block Mean	Estimation Bock Variance	Sum of the Positive Weights	KSOR	KE	Kriging Estimation Variance	Theoretical Block Variance
4	3.10	10.28	1.00	0.023	-2.73	11.89	3.20
8	3.18	7.70	1.00	0.032	-1.79	8.88	3.20
16	3.32	6.42	1.00	0.045	-1.10	6.70	3.20
32	3.42	4.80	1.00	0.061	-0.62	5.18	3.20
64	3.47	4.22	1.00	0.079	-0.35	4.30	3.20
128	3.59	3.49	1.00	0.098	-0.20	3.82	3.20
256	3.69	2.97	1.00	0.122	-0.11	nr	3.20
unique	5.00	0.32	1.00	0.244	0.01	nr	3.20

Table 3: Global statistics for the Minimal Information case estimates

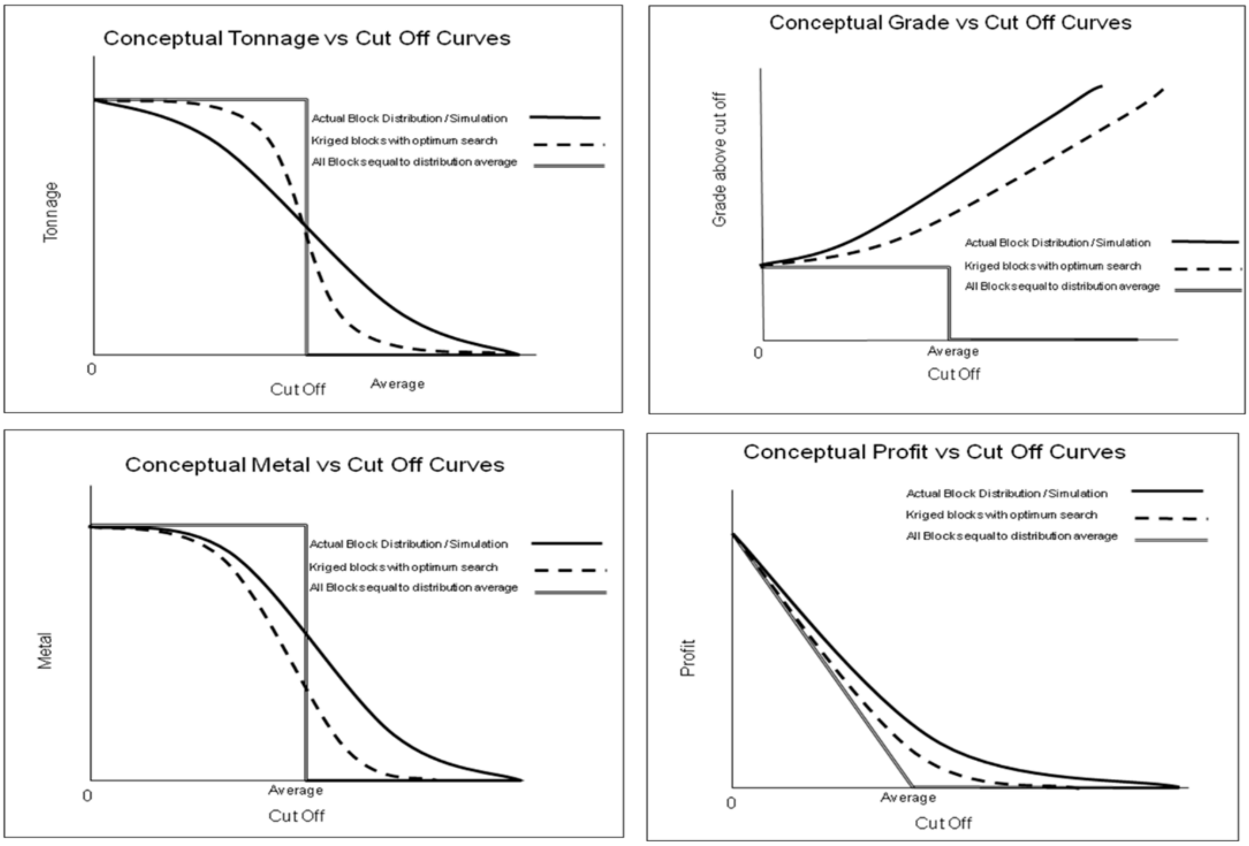


Figure 14: limits

Local versus global accuracy at extraction stage (final selection)

Performance (Figure 15) should not be confused with prediction. A final selection model's usefulness needs to be judged on how it performs, not how it predicts. This is where selectivity and elimination of conditional bias are critical. No model, however well informed, will ever match reality. The smoothing that results from attempts to eliminate conditional bias via maximisation of the KSOR is the key to achieving the best performance as it will 'balance' the mis-classification appropriately.

The example below shows the resulting performance of a model estimated from drill spacing approximately the same as bock size, compared to 'reality' derived from a model of the same block size created from drilling with between 4 to 8 holes within each block.

Another succinct quote from Chiles and Delfiner 1999 sums up the situation.

'For a high cut off grade [above zero] there will be a loss due to bad selection. Poor blocks are selected because estimated rich. Rich blocks are rejected because estimated poor. In all cases [except at zero cut off] this translates into a degradation of the value of the exploited ore [compared to prediction].'

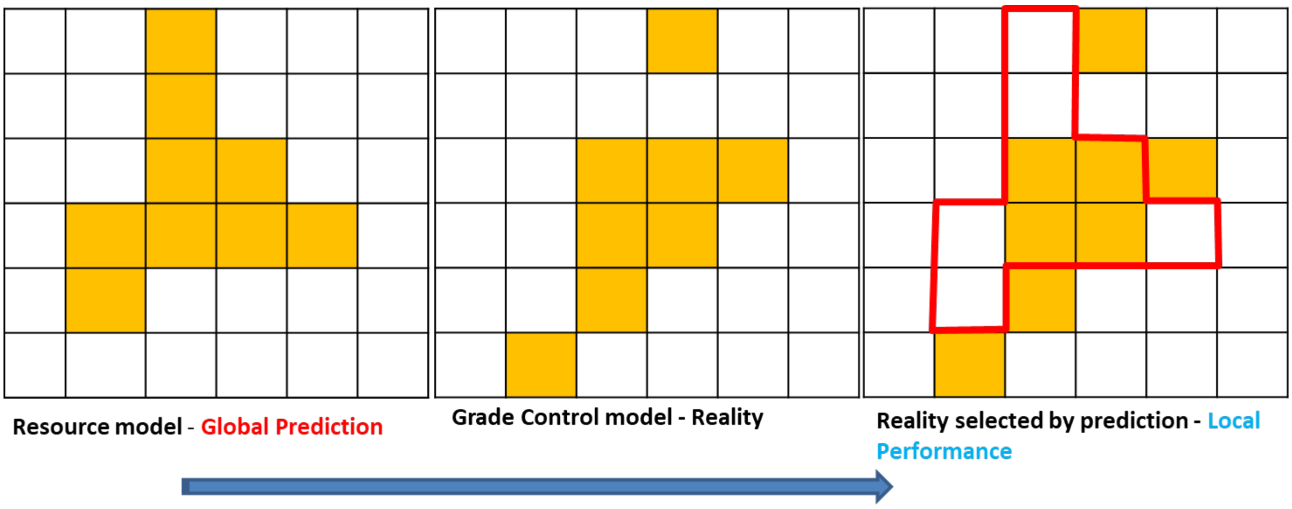


Figure 15: Prediction versus performance

For the Typical Information case we can see that, with the exception of the tonnage curve, the higher sample number estimates globally perform closer to the grade control model (Figure 16). This is particularly evident in the profit curves. This observation backs up our expected outcome, which is, the estimate with the lowest conditional bias, the highest KSOR (the smoothest estimate), is the best estimate for final selection both locally and globally. The converse also applies, targeting true block variability, is incorrect at final selection stage.

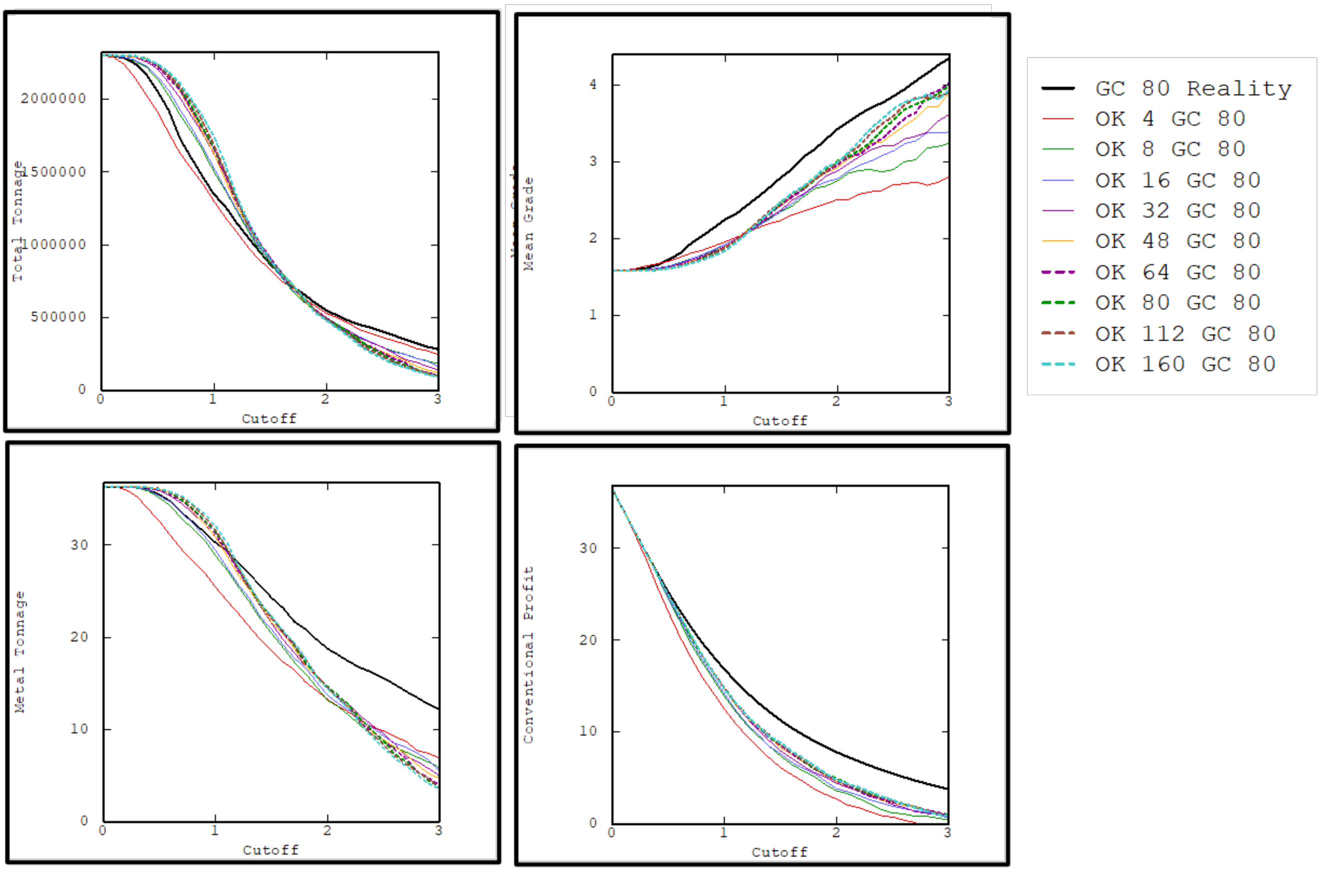


Figure 16: Performance curves

Classification at planning stage

Many practitioners use either the KSOR or the KE as part of the estimation quality aspect of classification procedures. This is often done in conjunction with other parameters such as distance to nearest sample, average drill spacing or kriging variance. Some practitioners like to run smoothing or closing algorithms or implicit models over the initial estimates at some selected threshold of KSOR or KE. Others will simply use the KSOR as a visual guide to create manual volumes in conjunction with other mathematical and/or geological parameters. Some practitioners will not use the KSOR at all and simple rely on drill spacing.

The above examples have shown how much the KSOR can vary in absolute terms depending on the variogram model, combined sample block geometry, and search neighbourhood parameters. However, the KSOR is still correct in each case because each case utilises a different set of parameters. The KSOR still has an objective meaning in each case.

One implication of this is that a classification based simply on drill spacing alone ignores the variogram and the search neighbourhood used. As we have demonstrated, a fixed drill spacing and variogram can have many different KSOR outcomes.

The problem that arises is that many practitioners, mostly because the software tells them to, target maximum regression slope by maximising sample numbers in the search neighbourhood for Resource estimate planning stage models. This has been demonstrated to produce models that may not be fit for purpose. These ‘unfit’ models may actually be classified with a higher confidence level (from a higher KSOR) than the models (which use fewer samples and therefore have a lower KSOR) that are actually fit for purpose.

If we have decided to target long-term planning global accuracy and have not maximised the regression slope, then we have apparently downgraded the quality of our local block by block estimate. This is correct, we have deliberately sacrificed local accuracy for global accuracy. So how should we interpret the KSOR for classification purposes? The answer, in part, is to interpret it as a relative measure, or alternatively, not use it at all.

Clearly, using a fixed threshold, say 0.9 for Measured will be problematic. Deposits could potentially be classified as locally Measured but globally Inferred!

Point 2 of the final item of Section 3 of table 1 of *the Australasian Code for Reporting of Exploration Results, Mineral resources and Ore Reserves* – The JORC Code 2012 Edition, under the criteria ‘Discussion of relative accuracy/confidence’, requires that;

‘The statement [of relative accuracy/confidence] should specify whether it relates to global or local estimates, and, if local, state the relevant tonnages, which should be relevant to technical and economic evaluation. Documentation should include assumptions made and procedures used.’

In practice, this item is rarely completed in any detail.

Some comments on variography

The KSOR is directly reliant on the variogram. As such, any uncertainty on the variogram is also an uncertainty on the KSOR.

Trends and zonal anisotropy

Zonal anisotropy is characterised by the total sill of an experimental variogram in a particular direction levelling out at a value significantly lower or higher than the total variance of the domain being examined. It typically reflects a distinct trend perpendicular to that direction. Trends tend to manifest themselves in a similar fashion in the experimental variogram but instead of plateauing they ‘drift’ gradually higher after a sharp rise (Figure 17). Both zonal anisotropy and directional drift confuse the concept of the total sill of a variogram because although they can be modelled as long-range structures, it can often be shown, by sub domaining, that the true local ranges are far shorter. Incorrectly fitting a long-range second or third structure to a domain that contains zonal anisotropy or a directional trend exaggerates the true continuity and in turn exaggerates the apparent KSOR that results during estimation. In the author’s experience, the estimated values themselves are often insensitive to fitting the of the total sill, due to the majority of samples being selected at short distances, but that the result of fitting a long-range structure to the total sill defined by the trend or zonal anisotropy is that the KSOR and KE can be significantly overstated, significant enough in some cases to for a change of classification to be required.

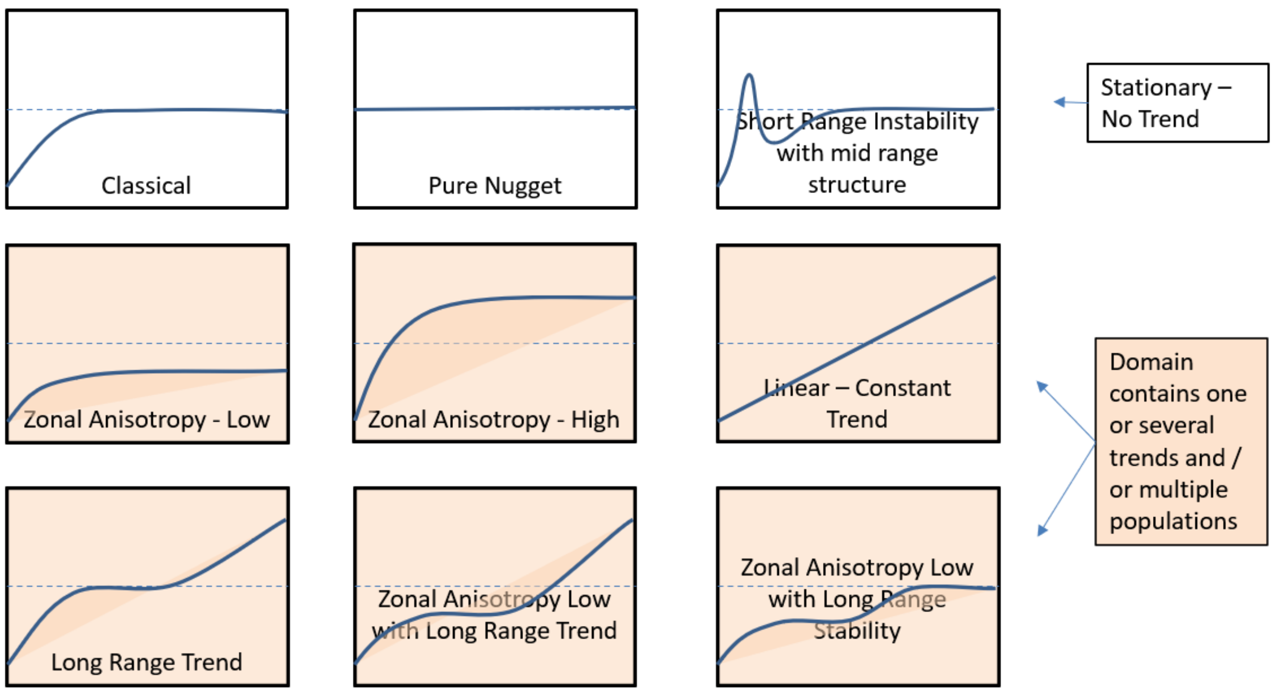


Figure 17: Types of experimental variogram encountered in practice

Empirical geostatistics – KNA

What parameters are we trying to target or reproduce?

How many samples constitute a 'good' neighbourhood? At the Resource stage (mine planning), what we are targeting when we refine our search neighbourhood is an SMU block distribution estimate that retains sufficient aspects of both local and global accuracy to enable reasonable estimates of global tonnages and grades that are likely to result from final selection, and at the same time provide sufficient broad-scale local accuracy to enable realistic modelling of potential mining scenarios.

KNA should be an iterative process. The first obvious target is that the domain mean is close to the de-clustered sample mean (de-clustering is an entire topic in itself). The second target is to minimise, but not eliminate, is negative weights. (Negative estimates should generally not occur, but small proportions of negative weights are allowable, even desirable in some circumstances). The third target is to approach the block variability and histogram shape observed in either final selection outcomes (if available) or a TCOS histogram. The fourth target is to understand the degree of smoothing occurring.

One way to examine the range of possible SMU histograms, without using a TCOS, is to simply use multiple ordinary kriging runs with a range of different search neighbourhood and high grade thresholding options.

If either production data or a TCOS model is not available then the technique referred to in Isaaks and Shrivastiva (1989) of reducing sample numbers until distinct variations are observed in the estimates is one useful approach.

A suggested set of steps for carrying out KNA is given below.

How to carry out KNA :

- Use the entire domain, not one block or a small group of blocks. In some cases, entire domains may have areas of very different drill spacing or contain significant zonal anisotropy or significant trends. In these cases, consideration should be given to carrying out KNA on separate sub domains.
- Determine the SMU size. Selection of the estimation block size (SMU) is a decision based on geology and mining method. We do not optimise block sizes during Resource KNA. The SMU size used for each analysis is therefore predetermined and fixed.
- Set the discretisation. Test theoretical block variance [C(v,v)] stability by randomising the discretisation locations. Increase discretisation points till the theoretical block variance is stable.
- Drill spacing. The data spacing at the time of analysis is what it is. We do not optimise sample spacing during Resource KNA. Drill spacing optimisation is a separate exercise.
- The fitted variogram at the time of analysis is what it is (but has an associated level of uncertainty). The variogram model is fixed.
- The only remaining choice is then the configuration of the search neighbourhood.
- Use sectors (typically octants or quadrants)
- Set the ellipse extents to very large, several times the variogram, ranges.
- Test a wide range of maximum sample numbers, for example 4, 8, 16, 24, 32, 48, 64, 80 and 128.
- For each test, and for each block, record the:
  - Estimated value
  - Sum of the positive weights (or some other measure of the negative weights)
  - KSOR
  - KE
  - Number of samples selected
  - Nearest sample distance
  - Mean sample distance

- Kriging variance

- Produce the grade, tonnage, metal and conventional profit curves for all test estimates.

Interpretation of KNA results:

The KNA and KSOR results are a function of sample spacing, block size, block discretisation, variogram model and search neighbourhood. Simply saying that small block kriging is good or bad or big block kriging is good or bad is nonsensical without taking the relative block size, sample spacing and variogram model configurations into account. It is more useful to consider three broad situations: 'Abundant Information', 'Typical Information' and 'Minimal Information' configurations.

What we are looking for is the extent of change in the grade tonnage etc. curves with the different maximum sample number scenarios. Table 4 sets out some criteria and observations for interpretation of KNA results.

	Abundant Information	**Typical Information**	**Minimum Information**
GT curves comparisons across all tests	GT curves are effectively the same	GT curves show moderate changes.	GT curves show massive changes
GT curves change at economic cut offs	<5%	Up to ~50%	Greater than ~50%
Negative weight behaviour	appear at relatively low sample numbers	appear at high sample numbers	may not occur at all
KSOR domain averages across all tests	KSOR typically > ~0.75	KSOR typically 0 to 0.75	KSOR typically ~= 0
KE domain averages across all tests	KE typically > ~ 0.5	KE typically between 0 and 0.5	KE typically -ve
TCOS	GT curves same as TCOS	TCOS curves will typically be near the lower sample end of the tests range	TCOS curves will typically fall somewhere in the middle of the test range
Can we use OK?	OK is good for SMU size	OK is good for SMU size if a low sample number neighbourhood is used. LUC, LMIK or some other panel-based method should be considered	OK cannot be used for SMU size. LUC, LMIK or some other panel-based method should be used or more samples should be obtained
Global accuracy (as compared to TCOS or production data)	Globally accurate across all tests	Globally accurate only if low sample number neighbourhood is used	Globally accurate only if low sample number neighbourhood is used and sufficient samples are available to infer the global histogram correctly
Local accuracy (as defined by KSOR = 1)	Can maximised by targeting maximum KE or KSOR and will be moderate to good	Can maximised by targeting maximum KE or KSOR with high sample neighbourhoods but will be poor to moderate	Local accuracy is non-existent.
Potential Classification from estimation quality perspective	Likely to be able to be classified as Measured or Indicated	Likely to be able to be classified as Measured or Indicated or Inferred	Likely to be able to be classified as Inferred or Unclassified
Recommended sample numbers for long term mine planning (Resource estimation)	Between 32 and 128 before negative weights become significant	Between 4 and 32 (or ¼ of the sample neighbourhood numbers that first produces significant weights)	Not recommended for mine planning. Use for scoping study or global grade and tonnage decisions only
Recommended sample numbers for final selection (Grade control)	Between 32 and 128 before negative weights become significant	Between 32 and 128 before negative weights become significant	Not applicable for final selection
If using LUC or LMIK or similar	NA	Analyse panel sized blocks in the same way	Analyse panel sized blocks in the same way

Table 4: Analysis of KNA results

Conclusions

SMU block size should be decided by reference to the geology and mining method, not by KNA or any estimation criteria.

Targeting a KSOR of 1 is only valid in two situations; final selection (grade control) and the Abundant Information case for Resource estimation. For the Typical Information Resource estimation case, the actual KSOR is not required to be maximised and KNA should seek to target grade and tonnage curves that match reality derived from existing production or from a rigorously defined TCOS model. For the Minimal Information case, where typically the appropriate SMU block size is significantly smaller than the data spacing and the effective variogram range is also short in relation to the average drill spacing, ordinary kriging should not be used. An alternative panel-based method such as LUC or LMIK which takes the volume variance effect into account should be used instead. The panels for this estimate should then be subject to KNA in the same manner as the SMU would be for OK. Ideally, in the Minimal Information case, more data should be acquired prior to estimation.

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